

QCD Thermodynamics with three degenerate Quarks on the Lattice

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for the RBC-Bielefeld Collaboration
— *first results from QCDOC* —

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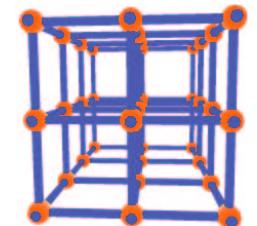
Christian Schmidt



OUTLINE

- 1) The Lattice Setup
 - ▶ the p4fat7-action
- 2) Preliminary Results
 - ▶ order parameters and susceptibilities
 - ▶ the critical temperature

C. Jung *et al.* [hep-lat/0509099]; M. Cheng *et al.* [hep-lat/0510035]



The Lattice Setup

— p4fat7-action

Adding irrelevant operators ($\xrightarrow[a \rightarrow 0]{} 0$):

Quarks:

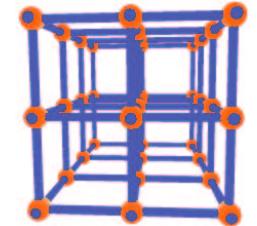
$$S_F(N_\tau, N_\sigma) = \sum_{n,n'} \sum_\mu \eta(n_\mu) \bar{\chi}_n \left(\frac{3}{8} \left[\frac{1}{1+6\omega} \left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \right) + \omega \sum_{\nu \neq \mu} \left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \right) \right] \right.$$

$$\left. + \frac{1}{48} \sum_{\nu \neq \mu} \left[\begin{array}{c} \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \text{---} \end{array} \right] \right) \chi_{n'} + \sum_n m_q \bar{\chi}_n \chi_{n'}$$

The coefficients are determined that way to make the free quark propagator rotational invariant up to $\mathcal{O}(p^4)$. → p4-action

[Karsch, Heller, Sturm (1999)]

(now p4fat3)



The Lattice Setup

— p4fat7-action

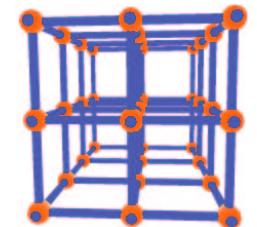
Incorporate MILC-type smearing:

$$\begin{aligned}
 X &= c_1 + c_3 + c_5 + c_7 \\
 &+ \text{Diagram } 6 \quad + \text{Diagram } 6x4 \quad + \text{Diagram } 6x4x2
 \end{aligned}$$

$c_1 = 1/8 \quad c_3 = 1/16 \quad c_5 = 1/64 \quad c_7 = 1/384$

- ▶ coefficients are chosen such that the couplings of quarks to gluons of momentum π/a are completely suppressed.

[Orginos, Toussaint, Sugar, 1999]



The Lattice Setup

— p4fat7-action

Gluons:

$$S_G(N_\tau, N_\sigma) = \sum_n \sum_{\mu, \nu > \mu} \left[\frac{5}{3} \left(1 - \frac{1}{3} \operatorname{Re} \operatorname{Tr} \begin{array}{c} \square \\ \square \end{array} \right) - \frac{1}{6} \left(1 - \frac{1}{6} \operatorname{Re} \operatorname{Tr} \left(\begin{array}{c} \square \\ \square \end{array} + \begin{array}{c} \square \\ \square \end{array} \right) \right) \right]$$

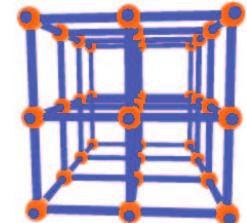
[Weisz, Wohlert (1984)]

- ▶ Symanzik improved
- ▶ eliminates the “cut-off” effects of order $\mathcal{O}(a^2)$

(tree-level improvement $\mathcal{O}(g^0)$)

The Lattice Setup

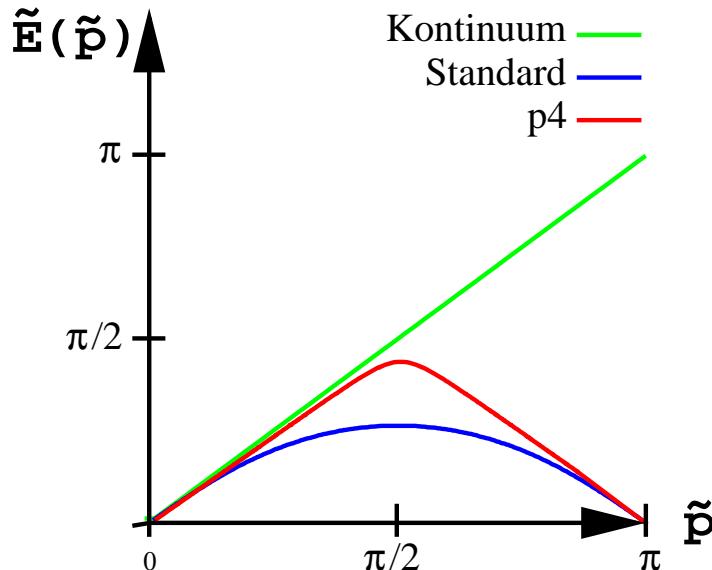
— p4fat7-action



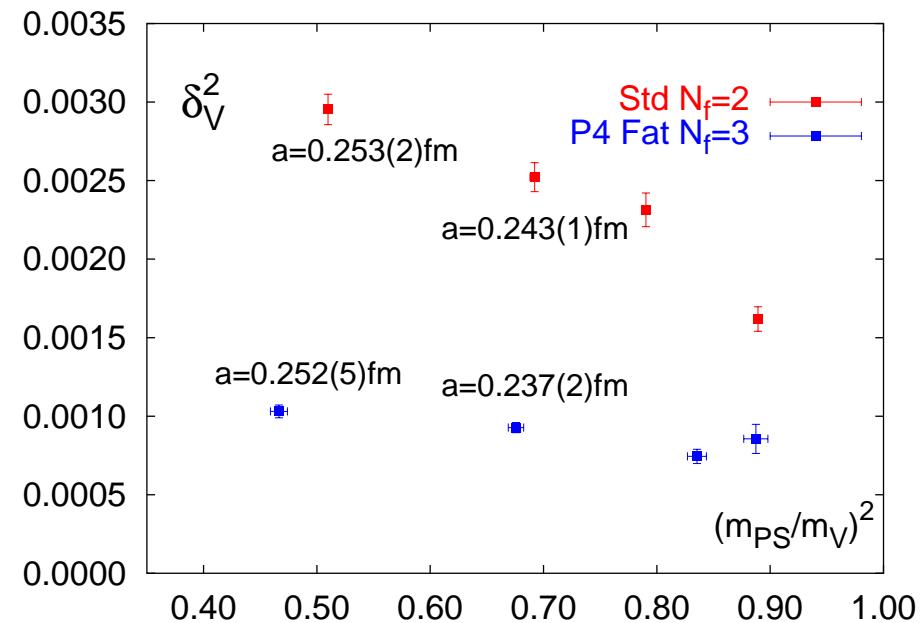
Properties of the p4-action:

- ▶ improved rotational symmetry of the free quark propagator: $\mathcal{O}(p^4)$
- ▶ improved flavor symmetry due to “fat-links”
(tree-level improvement: $\mathcal{O}(g^0)$)

dispersions-relation:

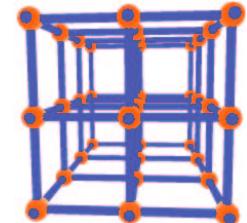


rotational symmetry of the heavy quark pot.:



The Lattice Setup

— p4fat7-action

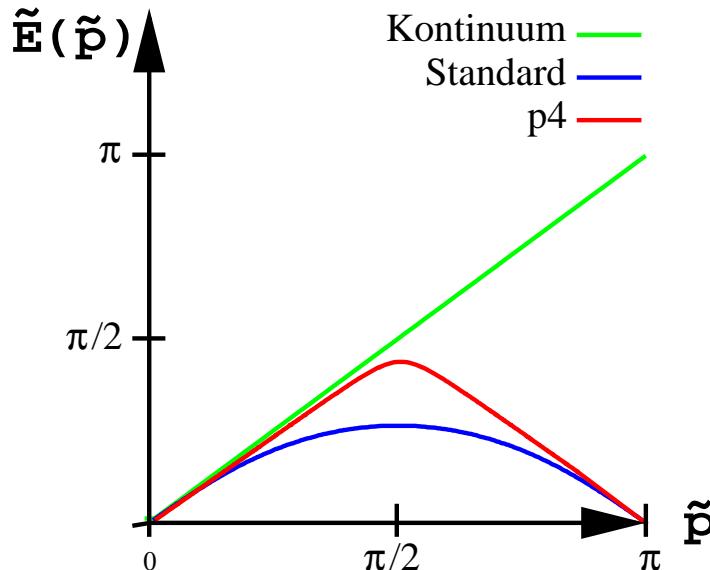


Properties of the p4-action:

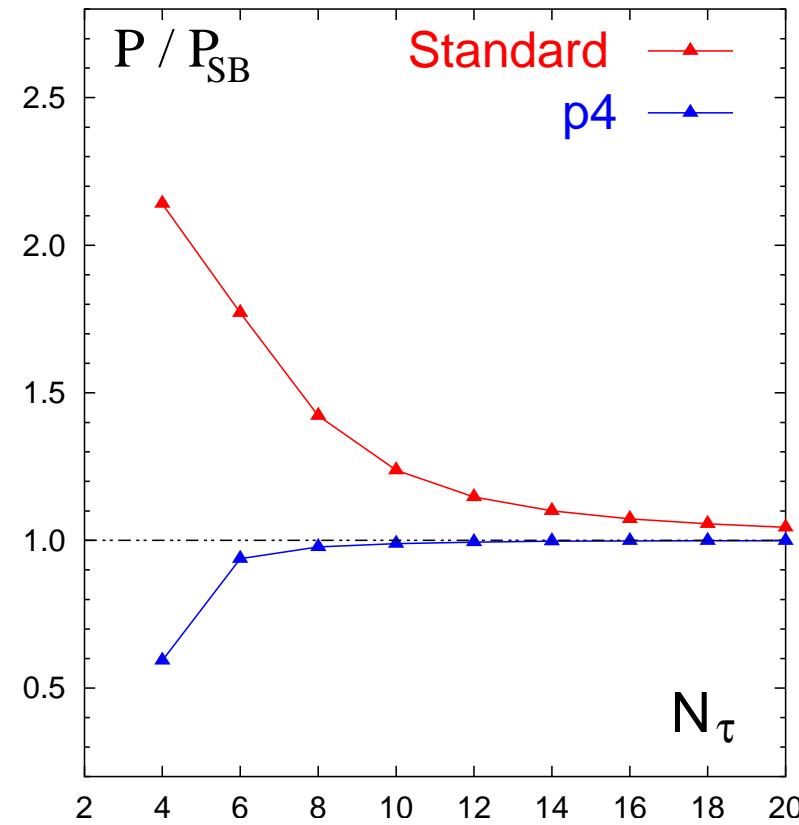
- improved rotational symmetry of the free quark propagator: $\mathcal{O}(p^4)$
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(tree-level improvement: $\mathcal{O}(g^0)$)

dispersions-relation:

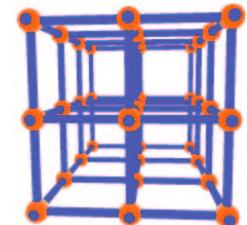


“cut-off” effects of the pressure:



The Lattice Setup

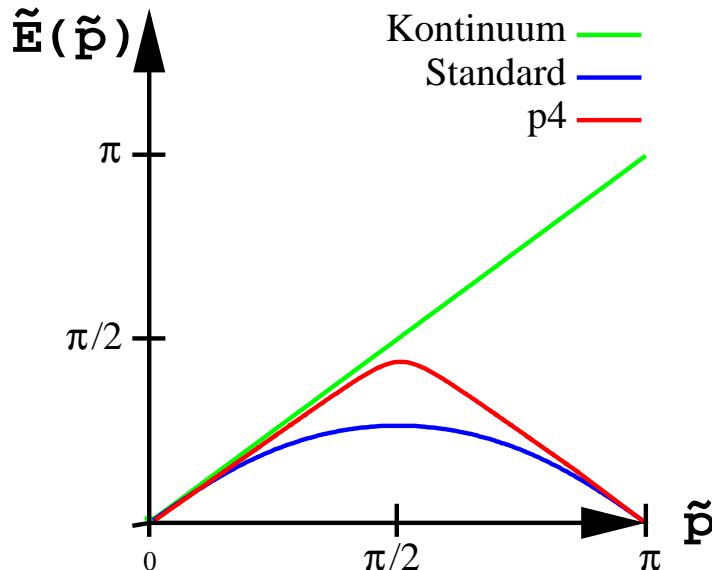
— p4fat7-action



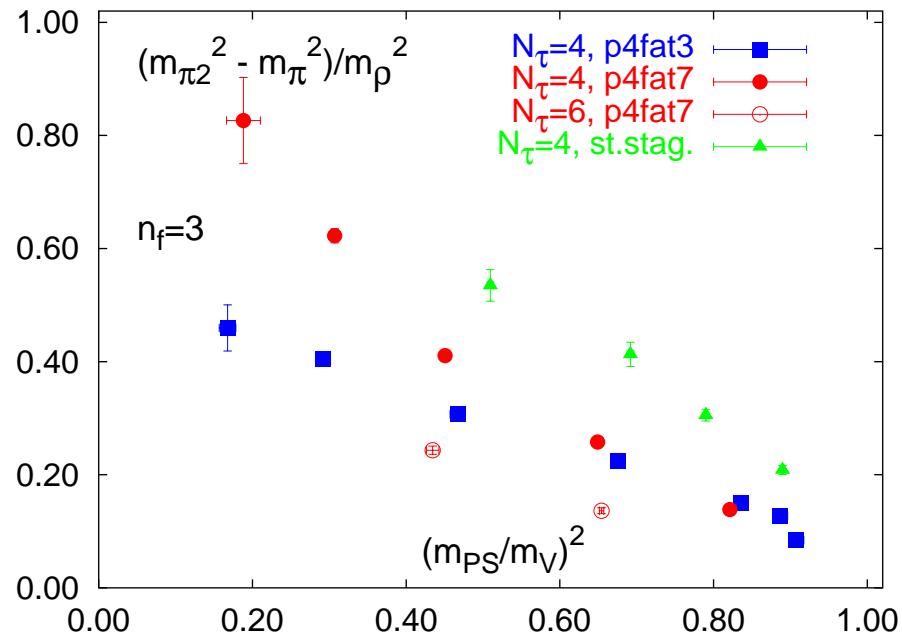
Properties of the *p4*-action:

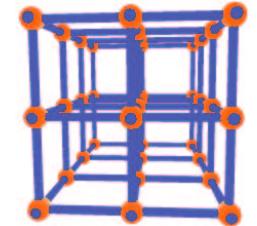
- ▶ improved rotational symmetry of the free quark propagator: $\mathcal{O}(p^4)$
- ▶ improved flavor symmetry due to “fat-links”
(tree-level improvement: $\mathcal{O}(g^0)$)

dispersions-relation:



non-Goldstone pions:





The Lattice Setup

— p4fat7-action

Goal:

- ▶ **almost realistic quark mass spectrum**

(2+1 flavor with a realistic strange quark, and almost realistic light quark masses,
 $m_\pi \approx 200 \text{ MeV}$)

- ▶ **exploring the continuum limit**

($a \approx 0.1 - 0.2 \text{ fm} \longrightarrow N_t = 4, 6, 8$)

- ▶ **analyzing the thermodynamic limit**

($V \lesssim 500 \text{ fm}^3 \longrightarrow N_s = 8, 16, 32$)

First Step:

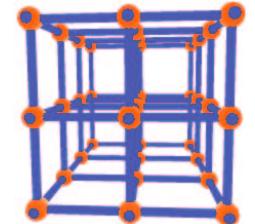
- ▶ **three flavor QCD**

($N_f = 3, m_{PS}/m_V \gtrsim 0.2$)

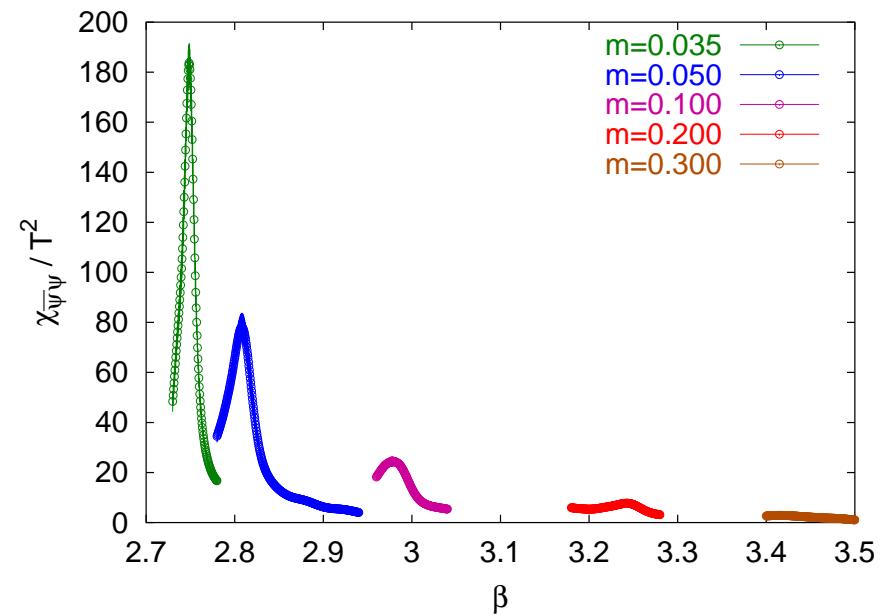
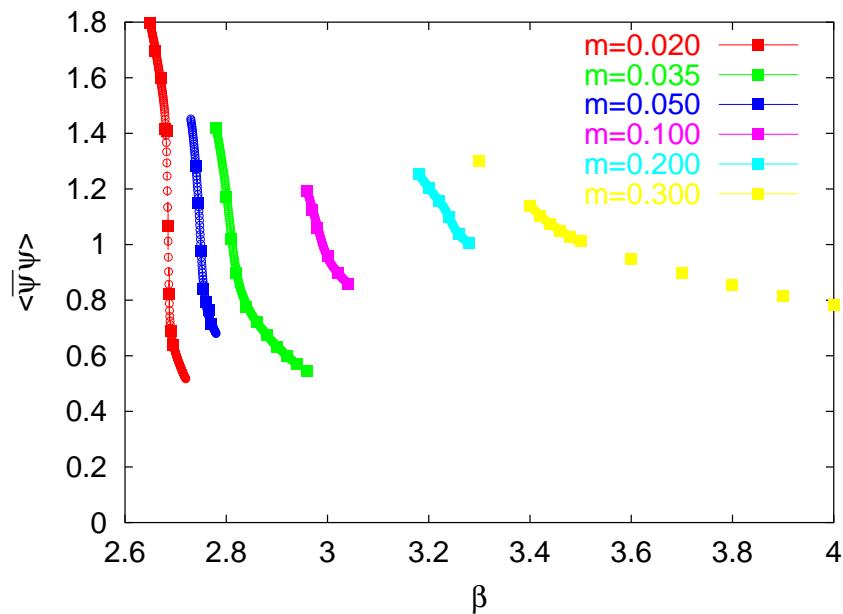
- ▶ **lattice spacings: $N_t = 4, 6$ and $N_s = 8, 16, 32$**

Preliminary Results

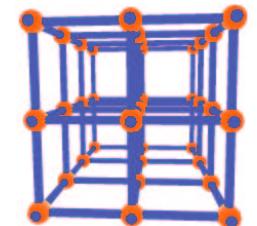
— Order Parameters and Susceptibilities



Chiral Condensate:



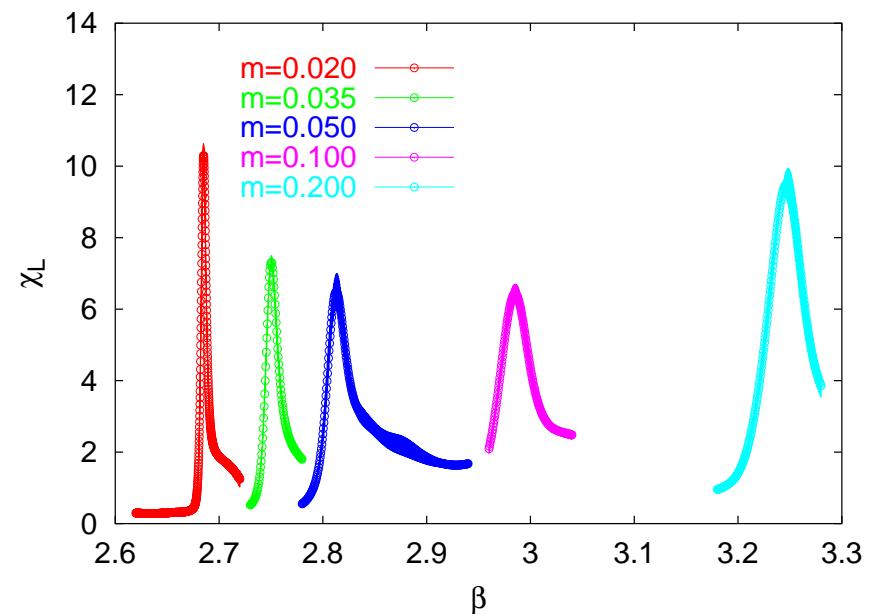
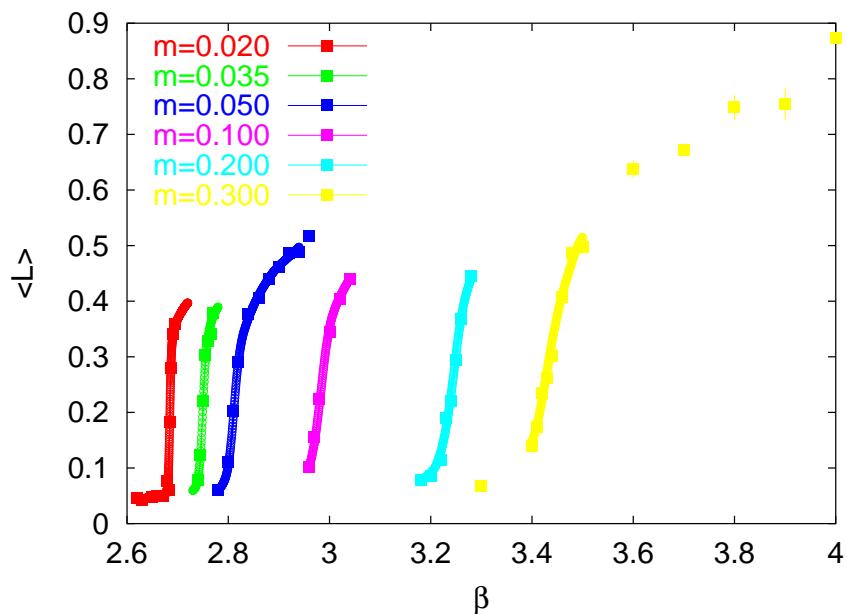
- ▶ a small beta separation enables a Ferrenberg-Swendsen analysis
- ▶ the transitions becomes stronger for m_q small
- ▶ critical couplings can be determined by the peak position of the susceptibilities



Preliminary Results

— Order Parameters and Susceptibilities

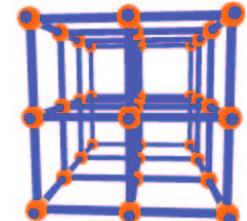
Polyakov Loop:



- ▶ a small beta separation enables a Ferrenberg-Swendsen analysis
- ▶ the transitions becomes stronger for m_q small
- ▶ critical couplings can be determined by the peak position of the susceptibilities

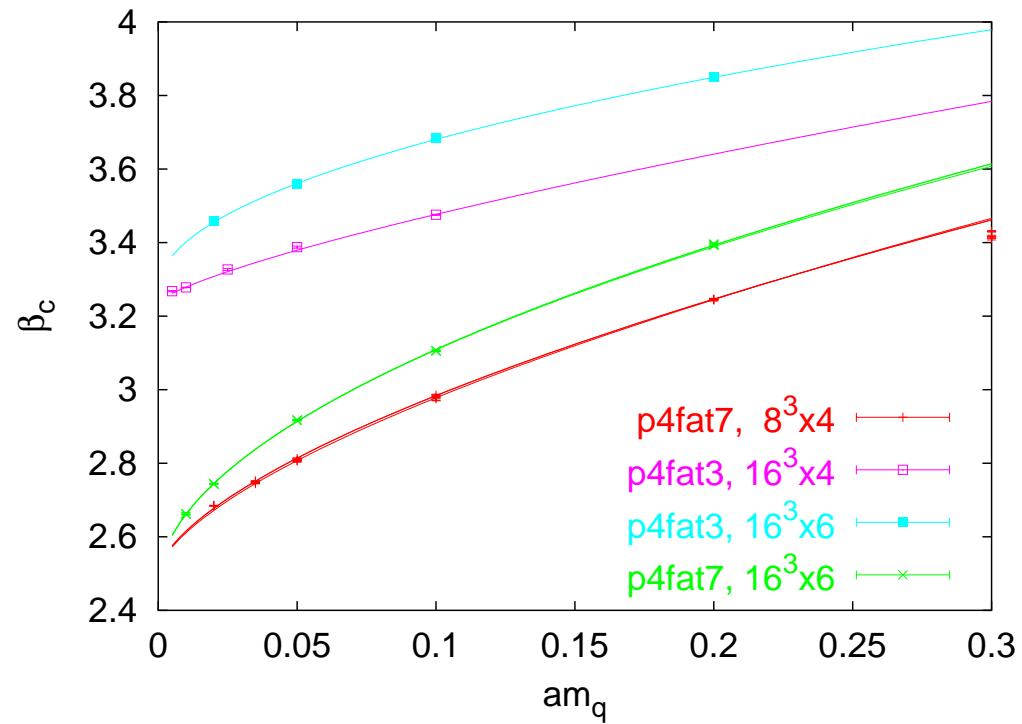
Preliminary Results

— The Critical Temperature



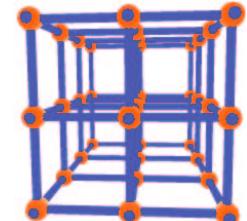
Critical Coupling β_c :

- ▶ the two actions show rather different critical couplings
- ▶ p4fat7 is shifted to smaller $\beta = 6/g^2$ (\rightarrow larger g)
- ▶ the beta separation:
 $\beta_c(N_t = 6) - \beta_c(N = 4)$
is small for p4fat7
($\rightarrow \partial a / \partial \beta$ large)
($\rightarrow \beta_c$ has to be determined very accurately)



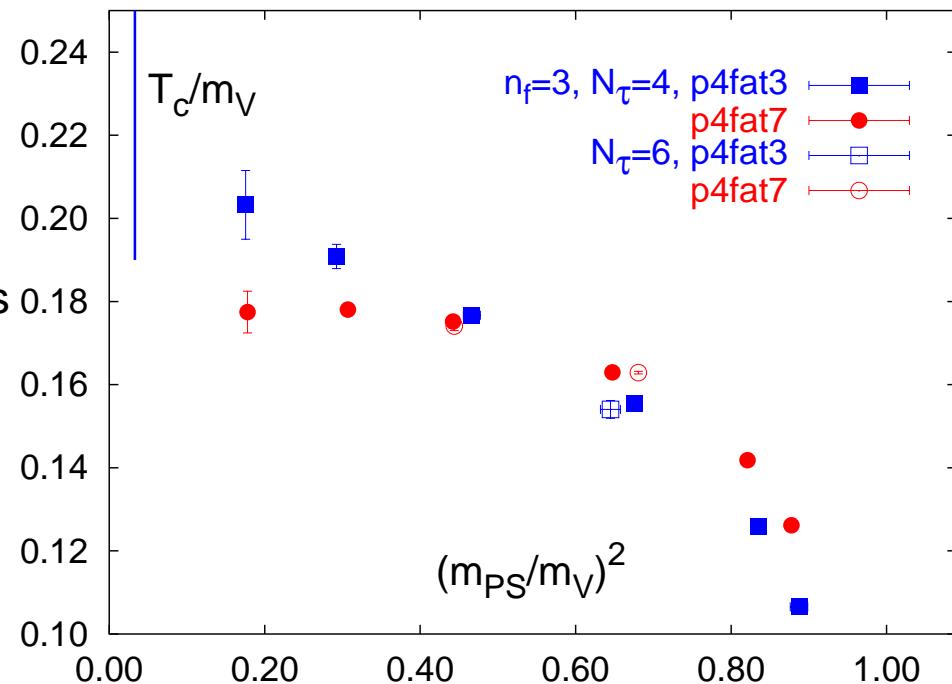
Preliminary Results

— The Critical Temperature



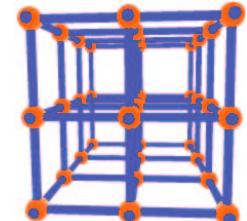
The Scale:

- ▶ calculate zero temperature hadron masses at β_c
e.g. $am_V(\beta_c(N_t))$
($\rightarrow T_c/m_V = 1/am_V(\beta_c(N_t))N_t$)
- ▶ consistency check between two actions
- ▶ p4fat actions show little cut-off effects
for $(m_{PS}/m_V)^2 \approx (0.4 - 1)$
- ▶ for $(m_{PS}/m_V)^2 \lesssim 0.4$ more statistics required
- ▶ at the moment $T_c \approx 145$ MeV favored
(in the chiral limit of $N_f = 3$)



Preliminary Results

— The Critical Temperature



Next Steps:

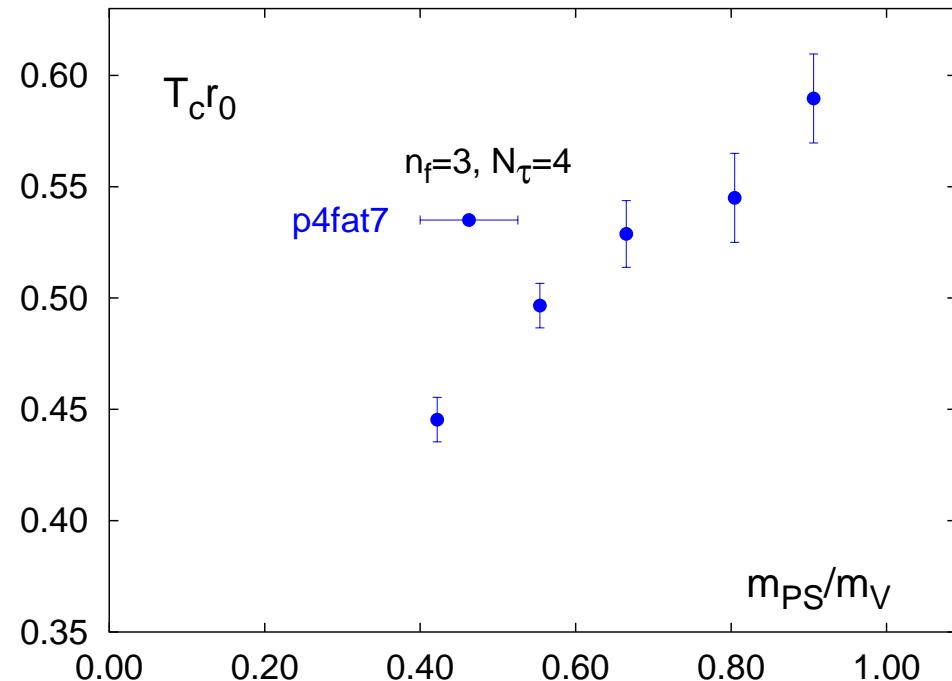
- ▶ improve scale calculation: heavy quark potential

$$r^2 (\partial V / \partial r) \Big|_{r=r_0} \equiv 1.65$$

$$r_0 = 0.5 \text{ fm}$$

- ▶ switch to (2+1)-flavor

- ▶ start calculation of equation of state



Goal:

- ▶ **almost realistic quark mass spectrum**

(2+1 flavor with a realistic strange quark, and almost realistic light quark masses,
 $m_\pi \approx 200 \text{ MeV}$)